Answers

1. A basis of $\ker A$ is for example $(-6, 4, 1, 0)$ and $(0, -1, 0, 1)$. A basis of $\text{im} A$ is for example $(1, 2, 0)$ and $(1, 3, 1)$.

2. The distance is $4/\sqrt{5}$. The closest point is $\frac{3}{5}(2, 1, 3, 4)$.

3. The maximum of $q$ on the unit sphere is 4 and is attained at $\pm \frac{1}{\sqrt{2}}(1, 0, -1)$. The minimum of $q$ on the unit sphere is $-2$ and is attained at $\pm \frac{1}{\sqrt{2}}(1, 0, 1)$.

4. $a_1 = 1$, $a_2 = 4$, $a_3 = 8$, $c = 1/9$. The plane of reflection is $2x_1 - x_2 - 2x_3 = 0$.

5. Page 27 in the lecture notes.

6. Three positive eigenvalues if $|a| < \sqrt{3}$ and two positive eigenvalues if $a \leq -\sqrt{3}$ or $a \geq \sqrt{3}$. 