No aids are allowed except the formula sheet provided in the examination hall and pens, pencils and erasers. The result will be posted at the latest on Wednesday, November 1 at 12:00.

1. Which of the following series are convergent?
   
a) \[ \sum_{k=1}^{\infty} \left( \frac{2 + i}{1 + 2i} \right)^k, \]
   b) \[ \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}, \]
   c) \[ \sum_{k=2}^{\infty} \frac{(-1)^kk}{k^2 - k}. \]

2. Find a solution \( u(x, t) \) to the following problem:
   \[
   \begin{aligned}
   \partial_t u(x, t) &= 3\partial_x^2 u(x, t), & 0 < x < \pi, \quad t > 0, \\
   u(0, t) &= u(\pi, t) = 0, & t > 0, \\
   u(x, 0) &= \sin 2x \cos 4x, & 0 < x < \pi.
   \end{aligned}
   \]

3. Let the function \( u \) be defined by
   \[ u(x) = \sinh x = \frac{e^x - e^{-x}}{2}, \quad -\pi < x \leq \pi. \]
   and \( u(x + 2\pi) = u(x) \) for any \( x \in \mathbb{R} \).
   a) Find the Fourier series of \( u \).
   b) Find the sum of the series
   \[ \sum_{n=1}^{\infty} \frac{n^2}{(n^2 + 1)^2}. \]

4. Find a power series solution \( u \) to the differential equation
   \[ xu''(x) + (1 + x)u'(x) + 2u(x) = 0, \quad u(0) = 1. \]
   Determine the radius of convergence and express \( u \) by means of elementary functions.

5. Find a function \( u \) such that
   \[ \int_{-\infty}^{\infty} u(x - y) e^{-|y|} dy = e^{-x^2/2} \]
   for all real \( x \).

6. Set
   \[ s(x) = \sum_{k=1}^{\infty} \frac{x}{1 + k^2 x^2}. \]
   a) Show that \( s(x) \) is convergent for all real numbers \( x \).
   b) Show that the function \( s \) is continuous for \( x > 0 \) and for \( x < 0 \).
   c) Is \( s \) continuous at 0?