1. Find the radius of convergence for the following power series:
   a) \( \sum_{k=1}^{\infty} \frac{k^2}{k^3 + 1} x^k \),
   b) \( \sum_{k=0}^{\infty} \left( \frac{1 + i}{1 + 2i} \right)^k x^k \),
   c) \( \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!} x^k \).

2. Find a solution \( u(x, t) \) to the following problem:
   \[
   \begin{align*}
   \partial_t u &= 2 \partial_x^2 u(x, t), \\
   \partial_x u(0, t) &= \partial_x u(\pi, t) = 0, \quad t > 0, \\
   u(x, 0) &= (\sin x)^4, \quad 0 < x < \pi.
   \end{align*}
   \]

3. Let the function \( u \) be defined by
   \[ u(x) = e^x, \quad 0 \leq x < 2\pi. \]
   and \( u(x + 2\pi) = u(x) \) for any \( x \in \mathbb{R} \).
   a) Find the Fourier series of \( u \).
   b) What is the sum of the series for \( x = 0 \)?
   c) Find the sum of the series
   \[ \sum_{n=1}^{\infty} \frac{1}{n^2 + 1}. \]

4. Determine a power series \( u(x) \) solving the problem
   \[ u''(x) - 2xu'(x) - 4u(x) = 0, \quad u(0) = 0, \quad u'(0) = 1, \]
   Determine the radius of convergence and express \( u(x) \) by means of elementary functions.

5. a) Find the Fourier transform of the function
   \[ u(x) = \frac{x}{1 + x^2}. \]
   b) Compute the integral
   \[ \int_{-\infty}^{\infty} \frac{x^2}{(1 + x^2)^2} dx. \]
6. Set

\[ s(x) = \sum_{n=1}^{\infty} \frac{nx}{1 + n^4x^2}. \]

a) Show that \( s(x) \) is convergent for all real numbers \( x \).

b) Show that the function \( s \) is continuous for \( x > 0 \) and for \( x < 0 \).

c) Is \( s \) continuous at 0?