**Answers**

1. a) \(R = 1\),  
   b) \(R = \frac{\sqrt{5}}{\sqrt{2}}\),  
   c) \(R = 4\).

2. The solution is  
   \[u(x,t) = \frac{1}{8} \left( 3 - 4e^{-8t} \cos 2x + e^{-32t} \cos 4x \right)\].

3. a) The Fourier coefficients of \(u\) are  
   \[c_n = \frac{e^{2\pi} - 1}{2\pi(1 - in)}\].

   b) The sum of the Fourier series for \(x = 0\) is \(\frac{e^{2\pi} + 1}{2}\).

   c)  
   \[
   \sum_{n=1}^{\infty} \frac{1}{n^2 + 1} = \frac{\pi(e^{2\pi} + 1)}{2(e^{2\pi} - 1)} - \frac{1}{2} = \frac{\pi \coth \pi}{2} - \frac{1}{2}.
   \]

4. The coefficients are given by the recursive equations  
   \[a_{k+2} = \frac{2a_k}{k+1} \implies a_{2k} = 0, \quad a_{2k+1} = \frac{1}{k!}\]  
   \[u(x) = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{k!} = xe^{x^2}\].

   The series is convergent for all \(x\).

5. a)  
   \[
   \hat{u}(\xi) = \begin{cases} 
   -i\pi e^{-\xi}, & \xi > 0, \\
   i\pi e^{\xi}, & \xi < 0.
   \end{cases}
   \]

   b) The integral is equal to \(\pi/2\).

6. The function \(s\) is not continuous at 0.